LRS Bianchi Type V Cosmological Model With Variable G and Λ

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Abstract: In this paper space-time varying gravitational constant G(x, t) and cosmological constant Λ(x, t) are explicitly determined as G(t) and Λ(t) in an LRS Bianchi type V model filled with perfect fluid satisfying barotropic equation of state. Physically realistic solutions of Einstein's field equations are obtained by assuming the condition 1). We observe that the corresponding models retain well established features of the standard cosmology, in addition expanding behaviours of the resulting model include late times accelerations in a particular situation.

Key Words: L.R.S. Bianchi type V, variable G and Λ.

1. INTRODUCTION

The two universal constants, G (gravitational constant) and Λ (cosmological constant), which appear in Einstein's field equations, if allowed to vary with time, have great significance, specially in the context of an expanding universe. Variations, either in G, or in Λ, or in both, have been a matter of interesting concern while trying to explain the nature of the expansion. Recent examples of such variations are available in the works of Bali and Singh [1]; Singh, Pradhan and Singh [2]; Pradhan and Kumar [3]; Prasad and Kumar [4]; and others. A positive Λ or a negative G contributes to the expansion whereas reverse is the situation for a positive G or negative Λ. The overall expansion (whether it is uniform or it is decelerated or accelerated) with the passage of time may be explained in terms of relative values of G and Λ. This requires in the first place that G and Λ must be determined explicitly.

Explicit determination of Λ and G has been achieved in this paper in the framework of an L.R.S. (Locally Rotationally symmetric) Bianchi type V space- time which is filled with perfect fluid. A particular solution of Einstein's field equations in this framework determines each of G(t) and Λ(t) if the cosmic fluid is allowed to obey a barotropic equation of state.

The scale factor **(S)** of the resulting model is arbitrary which we select for discussions conveniently. The usual power law form (viz., $S = t^n$, n>0) renders the model incompatible with the observational fact (vide works of Knop *et al.* [5]; Riess *et al.* [6]; Spergel *et al.* [7]; Tegmark *et al.* [8], and others) of late times accelerations. Instead, it provides a situation of uniformly decelerating expansion. Another specific form $S = (e^{nat} - 1)^{1/n}$, a>0, n>1; which corresponds to earlier work of Ellis and Madsen [9], puts the model in tune with above observational fact. The expansion in both the cases is isotropic throughout.

2. DERIVATION OF THE MODEL

We take an LRS Bianchi type V space-time in the form :

$$ds^2 = -dt^2 + A^2(x,t)\,dx^2 + B^2(x,t)\,e^{2x}(dy^2 + dz^2), \qquad (1)$$

which is filled with a co-moving perfect fluid.

The energy-momentum tensor T_{ij} of the fluid is given by

$$T_{ij} = (\rho + p)\,v_i v_j + p g_{ij}\,, \qquad (2)$$

in which ρ, p and v_i are respectively the energy density, the pressure and the unit flow vector of the fluid.

The Einstein's field equations:

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -8\pi G(x,t) T_{ij} + \Lambda(x,t)\, g_{ij} \qquad (3)$$

then lead to

$$8\pi G p = \frac{1}{A^2}\left[\left(\frac{B_1}{B}\right)^2 + \frac{2B_1}{B} + 1\right] - \left(\frac{B_4}{B}\right)^2 - \frac{2B_{44}}{B} + \Lambda \qquad (4)$$

$$8\pi G p = \frac{1}{A^2}\left[\frac{B_{11}}{B} - \frac{A_1 B_1}{AB} + \frac{2B_1}{B} - \frac{A_1}{A} + 1\right] - \frac{A_4 B_4}{AB} + \frac{A_{44}}{A} + \frac{B_{44}}{B} + \Lambda \qquad (5)$$

$$8\pi G \rho = \left\{\frac{-1}{A^2}\left[2\frac{B_{11}}{B} - 2\frac{A_1 B_1}{AB} - \frac{2A_1}{A} + \frac{6B_1}{B} + 3 + \left(\frac{B_1}{B}\right)^2 + \left(\frac{B_4}{B}\right)^2 + 2\frac{A_4 B_4}{AB} - \Lambda\right]\right\} \qquad (6)$$

and $$0 = \frac{B_{14}}{B} - \frac{A_4 B_1}{AB} - \frac{A_4}{A} + \frac{B_4}{B}, \qquad (7)$$

Vanishing divergence of T_{ij} leads to

$$0 = \rho_4 + (\rho + p)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right), \qquad 0 = p_1 \qquad (8)$$

whereas that of $(-8\pi G\, T_{ij} + \Lambda\, g_{ij})$ gives

$$0 = -8\pi(G_1 p - G_4 \rho) + 8\pi G\left\{-p_1 + \rho_4 + (p+\rho)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\right\} + \Lambda_1 + \Lambda_4, \qquad (9a)$$

in which

$$\Lambda_1 = 8\pi\,(G_1 p + G p_1), \qquad (9b)$$

and $$0 = \Lambda_4 + 8\pi\left[G_4 \rho + G\left\{\rho_4 + (\rho + p)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\right\}\right]. \qquad (9c)$$

The suffixes '1' and '4' in the above stand for differentiation with respect to x and t respectively.

Equations (8) and (9) simplify equivalently to

$$0 = \rho_4 + (\rho + p)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right), \qquad (10)$$

and $$\Lambda_4 + 8\pi G_4\, \rho = 0. \qquad (11)$$

Equations (4), (5), (6), (7), (10) and (11), when considered simultaneously, put the metric (1) in the form :

$$ds^2 = - dt^2 + e^{2g(t)}\,[dX^2 + e^{2sX}\,(dy^2 + dz^2)], \qquad (12)$$

in which $dX = e^k\, dx$ and $s = (m + 1)e^{-k}$.; where k and m are constants, the functions g(t) being arbitrary.

The form (12) then determines, for barotropic equation of state [$p = w\rho$, $0 \le w \le 1$], the quantities ρ, Λ and G as follows:

$$\ell . e^{(w \quad)g}, \qquad \ell \qquad (13)$$

$$\Lambda = \frac{2}{w+1}\left[\frac{(m+1)^2}{e^{2g+2k}} + g_{44}\right] - \frac{3(m+1)^2}{e^{2g+2k}} + 3g_4^2 \qquad (14)$$

and $$G = -\frac{e^{3(w+1)S}}{4\pi\ell(w+1)}\left[\frac{(m+1)^2}{e^{2g+2k}} + g_{44}\right]. \qquad (15)$$

3. DISCUSSIONS

The model (12) reduces, for m = – 1, to Friedmann – Robertson-Walker model with zero curvature. Also, it reduces to (1) for 0 $m+1 = e^k$. Leaving aside these two trivial cases, we discuss the model in two particular situations of the scale factor.

Firstly, when the scale factor has some positive power of t, i.e. when

$$S = e^{g(t)} = t^n , \; n > 0; \qquad (16)$$

we obtain the following expression :

Expansion scalar $\theta = v^i_{;i} = 3g_4 = \frac{3n}{t}$, (17a)

the semi-colon standing for covariant differentiation;

Shear = σ = 0 (identically), (17b)

Deceleration parameter $q = -\frac{SS_{44}}{(S_4)^2} = \frac{1}{n} - 1$ (17c)

$$\rho = l\, t^{-3(\omega+1)n} \quad , \qquad (17d)$$

$$\Lambda = \frac{2}{w+1}\left[\frac{(m+1)^2}{e^{2k}t^{2n}} - \frac{n}{t^2}\right] - \frac{3(m+1)^2}{e^{2k}t^{2n}} + \frac{3n^2}{t^2} \quad , \tag{17e}$$

$$G = \frac{t^{3n(w+1)}}{4\pi\ell(w+1)}\left[\frac{n}{t} - \frac{(m+1)^2}{e^{2k}t^{2n}}\right]. \tag{17f}$$

Each of θ and ρ, as above, tends to and 0 when t tends to 0 and respectively, whereas σ = 0 throughout. Thus the model starts with a bigbang at t = 0 and continues expanding isotropically till t = .

In the matter dominated case (w = 0), G → and - when t → 0 and respectively, provided n < 2/3. In-between, it takes the value zero at

$$t = \left[\frac{(m+1)^2}{ne^{2k}}\right]^{\frac{1}{2(n+1)}} \equiv t_0 \text{ (say)}.$$

Also, Λ → – and 0 respectively when t → 0 and . Thus initially, both G (>0) and Λ (<0) oppose the bigbang expansion till t = t_0, when G ceases to do so, while Λ continues. After t = t_0, G(<0) starts contributing to the expansion whereas Λ retains its earlier role, ofcourse with diminishing strength. But, the overall effect over the expansion is to decelerate it throughout uniformly, as is indicated by the constant value of q (= 1/n – 1) in which n < 2/3. In the radiation dominated (w = 1/3) situation, all the above discussions hold to be the same qualitatively but for n < ½.

Next, we choose the scale factor in a form which allows both the decelerated and accelerated phases of expansion. As done in our previous paper (Prasad *et al.* [4]),

we take the Hubble parameter (H) in the form

$$H = \frac{S_4}{S} = a\left(S^{-n} + 1\right) \quad , \tag{18}$$

in which a > 0 and n > 1 are constants. This is in the lines already set by Ellis and Madsen [9].

In view of (18) the function g(t) comes out to be

$$g(t) = \frac{1}{n}\log(e^{nat} - 1), \tag{19}$$

and hence the expressions corresponding to (17) are obtained as

$$\theta = 3a.\frac{e^{nat}}{e^{nat} - 1},$$

$$\sigma = 0,$$

$$q = \frac{n}{e^{nat} - 1},$$

$$\rho = \frac{\ell}{\left(e^{nat} - 1\right)^{3(\omega+1)n}} \,,$$

$$\Lambda = -\frac{(3w+1)}{(w+1)} \frac{(m+1)^2}{e^{2g+2k}} - \frac{2na^2.e^{nat}}{(w+1)(e^{nat} - 1)^2} + \frac{3a^2 e^{2nat}}{(e^{nat} - 1)^2} \,,$$

$$G = \frac{e^{3(w+1)S}}{4\pi\ell(w+1)} \left[\frac{na^2 e^{nat}}{\left(e^{nat} - 1\right)^2} - \frac{(m+1)^2}{e^{2g+2k}} \right].$$

This time, θ tends to and 3a when t tends to 0 and respectively, the corresponding limits for ρ being and 0. We have σ = 0 (again), which is really independent of S. So, the general set-up of the expansion is almost similar to that in the first case. But, Λ, G and q behave somewhat differently.

Deceleration parameter q is now a function of t and it tends to n – 1 when t → 0 and to – 1 when t → . In the matter dominated case (w=0), we find that G → and - when t → 0 and respectively, provided n = 2, k = 0, and $2a^2 < (m+1)^2$. Under the same restrictions over the constants, Λ → – and $3a^2$ respectively when t → 0 and . Thus Λ and G initially behave in ways similar to those in the first choice of the scale factor. But, their late time behaviours this time are really in tune with the observational fact of an accelerated universe. For, at

$$t = \frac{1}{2a} \log_e 2 = t^*$$

corresponding to q = 0, both Λ and G are positive and the two are equal provided further that $(m+1)^2 < 4a^2$ and $8\pi l = 1$. And beyond t = t*, each of and G tends to contribute to the expansion, which is in tune with the negative limits of q. All these go in favour of the observed (late times) accelerations of the real universe. In the radiation dominated case (w = 1/3), the above limiting values are the same except that G → $\frac{3a^2}{l}$ (finite) when t →0, and hence late times accelerations are also justified.

The model can be discussed similarly for other choices of the scale factor which must, of course, be chosen wisely.

4. CONCLUSIONS

A particular solution of Einstein's field equations containing variable G and yields explicit determination of these quantities in an LRS Bianchi type V universe filled with perfect fluid provided barotropic equation of state is satisfied. The resulting model has an arbitrary scale factor whose specification in a particular form renders the model allow late times accelerations. Other suitable specifications of the scale factor may reveal other interesting features of the model.

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